Notes for error analysis

1. **Limitations of Dimensional Analysis**
   The method of dimensions has the following limitations:
   (i) by this method the value of dimensionless constant cannot be calculated.
   (ii) by this method the equation containing trigonometric, exponential and logarithmic terms cannot be analyzed.
   (iii) if a physical quantity in mechanics depends on more than three factors, then relation among them cannot be established because we can have only three equations by equalizing the powers of M, L and T.
   (iv) it doesn't tell whether the quantity is vector or scalar.
2. **Significant Figures**
   The significant figures are a measure of accuracy of a particular measurement of a physical quantity.
   Significant figures in a measurement are those digits in a physical quantity that are known reliably plus the first digit which is uncertain.
3. **The Rules for Determining the Number of Significant Figures**
   (i) All non-zero digits are significant.
   (ii) All zeroes between non-zero digits are significant.
   (iii) All zeroes to the right of the last non-zero digit are not significant in numbers without decimal point.
   (iv) All zeroes to the right of a decimal point and to the left of a non-zero digit are not significant.
   (v) All zeroes to the right of a decimal point and to the right of a non-zero digit are significant.
   (vi) In addition and subtraction, we should retain the least decimal place among the values operated, in the result.
   (vii) In multiplication and division, we should express the result with the least number of significant figures as associated with the least precise number in operation.
   (viii) If scientific notation is not used:
   (a) For a number greater than 1, without any decimal, the trailing zeroes are not significant.
   (b) For a number with a decimal, the trailing zeros are significant.
4. **Error**
   The measured value of the physical quantity is usually different from its true value. The result of every measurement by any measuring instrument is an approximate number, which contains some uncertainty. This uncertainty is called error. Every calculated quantity, which is based on measured values, also has an error.
5. **Causes of Errors in Measurement**
   Following are the causes of errors in measurement:

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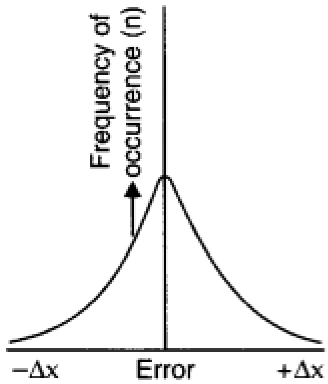

Illustration of Gaussian Law of Normal distribution.

Least Count Error. The least count error is the error associated with the resolution of the instrument. Least count may not be sufficiently small. The maximum possible error is equal to the least count.
Instrumental Error. This is due to faulty calibration or change in conditions (e.g., thermal expansion of a measuring scale). An instrument may also have a zero error. A correction has to be applied.
Random Error. This is also called chance error. It makes to give different results for same measurements taken repeatedly. These errors are assumed to follow the Gaussian law of normal distribution.
Accidental Error. This error gives too high or too low results. Measurements involving this error are not included in calculations.
Systematic Error. The systematic errors are those errors that tend to be in one direction, either positive or negative. Errors due to air buoyancy in weighing and radiation loss in calorimetry are **systematic errors.** They can be eliminated by manipulation. Some of the sources of systematic errors are:
(i) intrumental error
(ii) imperfection in experimental technique or procedure
(iii) personal errors

6. **Absolute Error, Relative Error and Percentage Error**

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– If $a_1, a_2, a_3, \ldots\ldots, a_n$ be the measured values of a quantity in several measurements, then their mean is considered to be the true value of that quantity i.e.,

$$\text{true value } a_0 = a_{\text{mean}} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

– The magnitude of the difference between the true value of the quantity and the individual measurement value is the absolute error of that measurement. Hence, absolute errors in measured values are:

$$\Delta a_1 = a_0 - a_1,\ \Delta a_2 = a_0 - a_2,\ \Delta a_3 = a_0 - a_3,\ \ldots\ldots\ldots\ldots,\ \Delta a_n = a_0 - a_n$$

– The arithmetic mean (*i.e.,* the mean of the magnitudes) of all the absolute errors is known as the mean absolute error.

$$\therefore \qquad \Delta a_{\text{mean}} = \frac{\left[|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|\right]}{n}$$

– The ratio between mean absolute error and the mean value is called relative error.

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Mean value}}$$

$$= \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} = \frac{\Delta a_{\text{mean}}}{a_0}$$

– Percentage error is the expression of the relative error in percentage.

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_{\text{mean}}} \times 100\%$$

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1. **Combination of Errors**
   - If a quantity Z be expressed as the sum or difference of two quantities A and B (*i.e.*, if $Z = A + B$ or $Z = A - B$), then maximum value of error $\Delta Z = \Delta A + \Delta B$.

     Hence, when two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors in the individual quantities.
   - If a quantity Z be expressed as product or a quotient of quantities A and B, then the maximum fractional error in Z is given by

     $$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

     Hence, when two quantities are multiplied or divided, the relative error in the result is the sum of the relative errors in the multipliers.
   - If $Z = A^m B^n C^l$ etc., then maximum fractional error in Z is given by

     $$\frac{\Delta Z}{Z} = m\frac{\Delta A}{A} + n\frac{\Delta B}{B} + l\frac{\Delta C}{C}$$