



CHAPTER 2

UNITS AND DIMENSIONS

1. Unit

To measure a physical quantity we require a standard of measurement. This standard is called the unit of that physical quantity. The measure of any physical quantity = nu , where n = numerical value of the measure of the quantity, u = unit of the quantity.

2. System of units

The common systems of units are (i) the foot-pound-second (fps) system, (ii) the centimeter-gram-second (cgs) system, (iii) the metre-kilogram second (MKS) system and (iv) the International System of Units (SI).

3. Base Units (Fundamental Units)

There are certain physical quantities which are very fundamental in nature. If the units of these quantities are defined, the units of all other quantities can be derived from these. Such quantities are called basic or fundamental quantities. The units of these quantities are called base units of the system. In mechanics three base units are required. These are units of length, mass and time. In heat and thermodynamics, standard of two more quantities are required. These are temperature and amount of substance. In electricity and magnetism a standard for current is required in addition to the three base units of mechanics. In light a standard for luminous intensity is required. Thus in all, in any self-consistent and well developed system of measurements, seven base units are required.

By international agreement the seven base units of the SI system are:

- (i) The metre (m) - standard of length
- (ii) the kilogram (kg) - standard of mass
- (iii) the second (s) - standard of time
- (iv) the ampere (A) - standard of electric current
- (v) the kelvin (K) - standard of temperature
- (vi) the candela (cd) - standard of luminous intensity
- (vii) the mole (mol) - standard of amount of substance

Properties of Base Units

Any standard unit should have the following two properties:



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(a) Invariability

The standard unit must be invariable. Thus, defining distance between the tip of the middle finger and the elbow as a unit of length is not invariable.

(b) Availability

The standard unit should be easily made available for comparing with other quantities.

Definition of base units

The metre (m): This is defined as 1650763.73 times the wavelength, in vacuum of the orange light emitted by in transition from $2p^{10}$ to $5d^5$.

The kilogram (kg): This is defined as the mass of a platinum-iridium cylinder kept at Sevres.

The second (s): This is the time taken by 9192631770 cycles of the radiation from the hyperfine transition in cesium - 133 when unperturbed by external fields.

The ampere (A): This is defined as the constant current which, if maintained in each of two infinitely long, straight, parallel wires of negligible cross-section placed 1 m apart, in vacuum, produces between the wires a force of 2×10^{-7} newton per meter length of the wires.

The Kelvin (K): In SI units, temperatures are measured on the thermodynamic scale with absolute zero as zero and the triple point of water (i.e., the temperature at which ice, water and water vapour are in equilibrium) as the upper fixed point. The interval is divided into 273.15 divisions and each division is taken as unit temperature. This unit is called the Kelvin.

The candela (cd): This is defined as the luminous intensity in the perpendicular direction of a surface of $1/600000$ square metre of a full radiator at the temperature of freezing platinum under a pressure of 101325 newtons per square meter.



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The mole (mol): The mole is the amount of any substance which contains as many elementary entities as there are atoms in 0.012 kg of the carbon isotope ^{12}C .

4. Prefixes, Multiplication Factors and their Units' Names in SI

Factor	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f



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10^{-18}	atto	a
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5. Derived units

Units of all other quantities may be obtained by some combination of the base units. These units are called derived units. Often derived units are given names. For example, the unit of force in SI units is kg ms^{-2} . It has been given the name Newton (N). The unit of power is $\text{kg m}^2 \text{s}^{-3}$. It has been given the name watt (W).

6. Some Important derived units

Through common usage, certain multiples and submultiples of the fundamental units have been given names. We will use them here and there but it is to be borne in mind that are not recognized in SI units.

Micron (mm) = 10^{-6} m

Angstrom ($\text{\AA}$) = 10^{-10} m

Fermi (fm) = 10^{-15} m

Barn (b) = 10^{-28} m²

7. Dimensions

The dimension of a physical quantity are the powers of the fundamental quantities to which they are to be raised to represent a unit of that physical quantity. The dimensions of fundamental quantities are expressed as (i) that of length by L, (ii) that of mass by M, (iii) that of time by T, (iv) that of current by I, (v) that of temperature by (K).

Symbolically, dimension of a physical quantity is written by putting that physical quantity within bracket such as [A] and it is read as dimension of A.

Some important physical quantities and their units with standard symbols:

Mass	kilogram	kg
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Length	metre	m
Time	second	s
Force	newton	N
Energy or work	joule	J
Power	watt	W
Current	ampere	A
Charge	coulomb	C
Potential Difference	volt	V
Resistance	ohm	Ω (omega)
Capacitance	farad	F
Magnetic flux	weber	wb
Magnetic induction field	tesla	T
Conductance	siemens	S
Temperature	kelvin or celsius	K or $^{\circ}\text{C}$



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Amount of substance	mole	mol
Luminous intensity	candela	cd
Illuminance	lux	lx
Luminous flux	lumen	lm
Frequency	hertz	Hz

Application 1

Find the dimensions of the following quantities:

(i) acceleration

(ii) angle

(iii) density

(iv) kinetic energy

(v) constant of gravitation

(vi) permeability of medium.

Solution:

(i) Acceleration = $\frac{\text{Velocity}}{\text{Time}}$, $[\text{Acceleration}] = [\text{Velocity}][\text{Time}] = \text{LT}^{-1}\text{T} = \text{LT}^{-2}$

(ii) Angle = $\frac{\text{Distance}}{\text{Distance}}$ i.e Angle is dimensionless.

(iii) Density = $\frac{\text{Mass}}{\text{Volume}}$, $[\text{Density}] = [\text{Mass}][\text{Volume}] = \text{ML}^3 = \text{ML}^{-3}$

(iv) Kinetic energy = $\frac{1}{2} \text{Mass} \times \text{Velocity}^2$, $[\text{Kinetic energy}] = [\text{Mass}] [\text{Velocity}]^2 = \text{ML}^2\text{T}^{-2}$

(v) Constant of gravitation occurs in Newton's law of gravitation

$$F = G \frac{m_1 m_2}{d^2}$$

$$[G] = [F][d^2][m_1][m_2] = \text{MLT}^{-2}\text{L}^2\text{MM} = \text{M}^{-1}\text{L}^3\text{T}^{-2}$$

(vi) Permeability occurs in Ampere's law of force

$$\Delta F = \mu (i_1 \Delta l_1)(i_2 \Delta l_2) \sin \theta / r^2$$

$$[\mu] = [\Delta F][r^2][i_1 \Delta l_1][i_2 \Delta l_2] = \text{MLT}^{-2}\text{L}^2\text{ILIL} = \text{MLT}^{-2}\text{I}^{-2}$$

8. Uses of dimensions

(i) To change values of physical quantity from one system into another.

(ii) To test the corrections of the results already arrived at



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(iii) To determine the exact relation between physical quantities which are likely to be interlinked.

Change from one system to another

Application 2

The value of a force on a body is 20 N in SI units. What is the value of this force in cgs units, that is, dynes?

Solution

Dimension of force = MLT^{-2}

Unit of force in SI = $kg\ m\ s^{-2}$

Unit of force in cgs = $g\ cm\ s^{-2}$

Let n be the numerical value of the force in cgs system.

$20\ kg\ m\ s^{-2} = n\ g\ cm\ s^{-2}$

$n = 20(kg)(m)(s^{-2}) = 20(1000g)(100cm)(s^{-2})$

20 newtons = 2×10^6 dynes

Checking correctness of an equation

Application 3

Check by the method of dimensions whether the following relations are true.

(i) $t = 2\pi l g^{-1} \sqrt{\dots}$. (ii) $v = PD^{-1} \sqrt{\dots}$, where v = velocity of sound and P = pressure. D = density of medium.

(iii) $n = \frac{1}{2l} \sqrt{\frac{F}{m}}$ where n = frequency of vibration l = length of the string, F = stretching force, m = mass per unit length of the string.

Solution



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$$(i) [R. H. S.] = [L][g] \sqrt{L} = L^2 T^{-2} \sqrt{L} = L^{\frac{5}{2}} T^{-2}$$

$[L. H. S.] = [t] = T$. Hence the relation is correct.

$$(ii) [R. H. S.] = [P][D] \sqrt{ML^{-1}T^{-2}ML^{-3}} = L^2 T^{-2} \sqrt{M^2 L^{-4} T^{-2}} = L^2 T^{-2} M T^{-1} L^{-2} = M T^{-3}$$

$[L. H. S.] = [v] = LT^{-1}$. Hence the relation is true.

$$(iii) [R. H. S.] = 1/[I][F][m] \sqrt{LMLT^{-2}ML^{-1}} = 1/[I][F][m] \sqrt{L^2 MT^{-2}} = 1/[I][F][m] L T^{-1} M^{\frac{1}{2}}$$

$$[L. H. S.] = [A \text{ Number Time}] = T^{-1}$$

Hence the result is correct.

Determining the exact relation between physical quantities

Application 4

Assuming that the critical velocity of flow of a liquid through a narrow tube depends on the radius of the tube, density of the liquid and viscosity of the liquid, find an expression for critical velocity.

Solution

Suppose, $v = k r^a \rho^b \eta^c$ where r = radius of the tube, ρ = density of liquid, η = coefficient of viscosity, k is a dimensionless constant and a , b and c are the unknown powers to be determined.

$$[v] = [r]^a [\rho]^b [\eta]^c$$

$$\text{or } LT^{-1} = L^a (ML^{-3})^b (ML^{-1}T^{-1})^c = M^{b+c} L^{a-3b-c} T^{-c}$$

Equating power of M , L and T we have

$$b + c = 0$$

$$a - 3b - c = 1$$

$$\text{and } -c = -1$$

$$\text{Hence } c = 1, b = -1, a = -1$$

$$v = k r^{-1} \rho^{-1} \eta^1 \text{ or } v = k \eta / \rho r$$